Optimal Procurement Auction for Cooperative Production of Virtual Products

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Abstract

We set up a supply-side game-theoretic model for cooperative production of virtual products. In our model, a group of producers collaboratively produce a virtual product by contributing costly input resources to a production coalition. Producers are capacitated, i.e., they cannot contribute more resources than their capacity limits. Our model is an abstraction of emerging internet-based business models such as federated learning and crowd computing. To maintain an efficient and stable production coalition, the coordinator should share with producers the income brought by the virtual product. Besides the demand-side information asymmetry, another two sources of supply-side information asymmetry intertwined in this problem: 1) the capacity limit of each producer and 2) the cost incurred to each producer. In this paper, we rigorously prove that a supply-side mechanism from the VCG family, PVCG, can overcome such multiple information asymmetry and guarantee truthfulness. Furthermore, with some reasonable assumptions, PVCG simultaneously attains truthfulness, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balancedness on the supply side, easing the well-known tension between these four objectives in the mechanism design literature.

Introduction

On this flat earth created by the internet, hierarchies of companies are falling away. Production activities are no longer a process confined within the border of an enterprise. Independent businesses can cooperate seamlessly through distributed computing systems to produce valuable virtual products. For example, a fast-developing technology, federated learning (FL), enables businesses to train (produce) artificial intelligence models collaboratively (Yang et al. 2019a). Another example is crowd computing, where participants contribute their redundant computing capacity to distributed computing tasks (e.g., protein folding simulation in the Folding@home project (Beberg et al. 2009)). These production models differ from classical ones in at least two respects: First, collaborative production creates synergies, i.e., participants working together generate more value than they working separately; Second, the output is virtual products, which is non-tangible and non-rivalrous, i.e., the consumption of the virtual products by one participant does not reduce the amount available for others. While the first feature encourages cooperation, the second feature results in the free rider problem, i.e., participants lack the incentives to ally to attain the socially optimal result. Currently, cooperative production applications are mostly run as small-scale not-for-profit projects, heavily relying on volunteers' participation, thus hindering their popularization. A profitable business model is necessary for cooperative production to develop faster in the economic world. According to traditions in the economic literature, we analyze the demand for output products and the supply of input resources separately: on the demand side, we need to charge consumers for their usage of the virtual products and maximize the income of the coalition; on the supply side, we should pay participating producers for their contribution of input resources. The demand-side sub-problem can be solvded by existing game-theoretic studies, e.g, we can use the Crémer-McLean mechanism (Crémer and McLean 1985; Kosenok and Severinov 2008) to extract full consumer surplus, but the supply-side sub-problem has not received enough attention yet.

In this paper, we set up a game-theoretic model for cooperative production of virtual products, based on which we propose a supply-side mechanism which determines payments to producers. Our contributions include: 1) We set up a game-theoretical model for cooperative production of virtual products, which takes into consideration

*This work was supported by LogiOcean Technologies, Ltd.

twofold supply-side information asymmetry in addition to the demand-side information asymmetry; 2) we propose a procurement auction that incentivizes producers to report their private information honestly (truthfulness) and maximizes social welfare ex post (allocative efficiency); 3) We apply neural network methods to optimizing for ex-post individual rationality and ex-post weak budget balancedness; 4) we prove with some reasonable assumptions, truthfulness, ex-post allocative efficiency, ex-post individual rationality and ex-post weak budget balancedness can be attained simultaneously on the supply side.

Related Works

Our game-theoretic model is an abstraction of emerging internet-based cooperative production practices. Although underlying technologies of cooperative production are fast developing, existing studies on these new business models are quite limited. From a more general perspective, our model is a special type of cooperative games (Peleg and Sudhölter 2007; Branzei, Dimitrov, and Tijs 2008; Chalkiadakis, Elkind, and Wooldridge 2011), which have attracted great attention from the algorithmic game theory community (Sandholm and Lesser 1997; Conitzer and Sandholm 2006; Chalkiadakis et al. 2010; Elkind et al. 2009; Bachrach et al. 2009). Designing an optimal sharing rule is a key concern when studying cooperative games (Kronbak and Lindroos 2007; Niyato, Vasilakos, and Kun 2011; Weikard 2009).

We decouples the cooperative production game into a demand-side game and a supply-side game. On the demand side, we use the Crémer-McLean mechanism (Crémer and McLean 1985; Kosenok and Severinov 2008; Albert, Conitzer, and Lopomo 2015) to extract consumer surplus. On the supply side, we proposed the PVCG mechanism, which borrows the idea of procurement auction from procurement games (Chen et al. 2005; Chandrashekar et al. 2007; Iyengar and Kumar 2008; Drechsel and Kimms 2010). Compared to existing research, our model takes into account three sources of information asymmetry, one on the demand side and two on the supply side, yet we simultaneously achieve truthfulness, allocative efficiency, individual rationality, and ex-post weak budget balancedness, easing the well-known tension between these objectives (Jackson 2014). We also follow the recent trend of neural-network-based mechanism design (Shen, Tang, and Zuo 2019).

Federated learning (Yang et al. 2019a; 2019b) and crowd computing (Beberg et al. 2009; Larson et al. 2009) are examples of the cooperative production game under study. Therefore, our work is also related to the literature on training machine learning models with strategic participants (Jia et al. 2019; Wang 2019; Cai, Daskalakis, and Papadimitriou 2015; Richardson, Filos-Ratsikas, and Faltings 2019; Westenbroek et al. 2019; Yu et al. 2020) and crowd computing from the angle of game theory (Christoforou et al. 2013).

Game Settings

We study a cooperative production game where a set of n *producers*, denoted by $N = \{0, 1, \ldots, n-1\}$, cooperatively produce a valuable *virtual product*, of which copies are dilivered to m *consumers*, denoted by $M = \{n, n+1, \ldots, n+m-1\}$. A participant may be both a producer and a consumer, but we assume that its behaviors as a producer and as a consumer are independent. On the *supply side*, producers contribute input resources to the production coalition, e.g., labor, raw materials, equipments, etc. On the *demand side*, consumers are granted access to utilize the output virtual product. We introduce a parameter $x_i \geq \mathbf{0}$ to measure the input resources contributed by producer i. x_i may be a vector when multiple input resources are involved. We use another parameter y to measure the *usefulness* of the output product, which is determined by the contributed input resources from all producers, i.e., y is a function of $\boldsymbol{x} = (x_0, \ldots, x_{n-1})$.

The usefulness of the output product determines the value it brings to consumers. We use the parameter $v_j, j \in N$ to denote the valuation of participant $j \in M$ on the output product. v_j is a function of y and a type parameter θ_j (called *valuation type*) that reflects the heterogeneity among consumers. We denote this function by $w(\cdot)$ and call the composite function of $w(\cdot)$ and $y(\cdot)$ the *individual valuation function*, denoted by $v(\cdot)$, i.e., $v(\boldsymbol{x}, \theta_i) = w(y(\boldsymbol{x}), \theta_i) = v_i$. For convenience, θ_j is such chosen that $v(\boldsymbol{x}, \theta_j) \equiv 0$ when $\theta_j = 0$. We denote $\boldsymbol{\theta} = (\theta_n, \theta_{n+1}, ..., \theta_{n+m-1})$. Contributing resources to the cooperative production process incurs costs to participants. Producer i's cost $c_i = c(x_i, \gamma_i)$ is a function (called *individual cost function*) of x_i and another type parameter γ_i that reflects the heterogeneity of producer i. We denote $\boldsymbol{\gamma} = (\gamma_0, ..., \gamma_{n-1})$. In our game, producers are assumed to be capacitated, i.e., producer i cannot contribute more resources than its capacity limit $\bar{x}_i$, i.e., $x_i \leq \bar{x}_i$. Both the type parameters γ_i, θ_j and the capacity limit $\bar{x}_i$ are private information unknown to the coalition coordinator a priori. This coordinator makes the transfer payment p_i to producer i and p_j to consumer j. Participants' preference is represented by quasi-linear utilities $u_i = p_i - c_i$, $i \in N$ and $u_j = p_j + v_j$, $j \in M$. We use *social surplus* $S(\boldsymbol{x}, \boldsymbol{\gamma}) = \sum_{j=n}^{n+m-1} v(\boldsymbol{x}, \theta_j) - \sum_{i=0}^{n-1} c(x_i, \gamma_i)$ to measure the social effect of the production coalition. Social surplus maximization implies *Pareto efficiency* (or *allocative efficiency* in the language of mechanism design).

Parameters in our model have concrete meanings in scenarios such as federated learning (FL) and crowd computing. In FL, the input resources are data. x_i measures the size and quality of the contributed dataset. (Measuring the quality of datasets is a separate problem. We do not go deeper here.) The capacity limit $\bar{x}_i$ is the best dataset owned by participant i. c_i is the cost of collecting and cleaning data. v_i is the value of the federated model to each participant (e.g., for the use case that banks use FL to train AI models to predict credit risk, v_i is calculated as the reduced bad debt rate times the principle of loans). In crowd computing, x_i is the contributed computing power, c_i costs of computing power (e.g., electricity costs, hardware costs, etc), and v_j is the value of the computation result (e.g., the value of protein folding result to drug development).

We put forward five assumptions on the individual valuation function $v(\boldsymbol{x}, \theta_j)$ and the individual cost function

$c(x_i, \gamma_i)$. Assumption 1-2 are used for proving truthfulness and alloactive efficiency. Assumption 3-5 are used for proving individual rationality and ex-post weak budget balancedness.

Assumption 1 (Smoothness and monotocity)**.** *The individual valuation function $v(\boldsymbol{x}, \theta_j)$ is a smooth and monotonic increasing function of $\boldsymbol{x}$ and θ_j. The individual cost function $c(x_i, \gamma_i)$ is a smooth and monotonic increasing function of x_i and γ_i.*

Assumption 2 (Zero input resource makes no difference)**.** *If $x_i = \mathbf{0}$, then: (1) participant i makes no difference to the value of the output product, i.e., $v(\boldsymbol{x}, \theta_j) = v((\mathbf{0}, \boldsymbol{x}_{-i}), \theta_j) = v(\boldsymbol{x}_{-i}, \theta_j), \forall j, \theta_j$; (2) participant i bears no cost, i.e., $c(x_i, \gamma_i) = c(\mathbf{0}, \gamma_i) = 0$, $\forall \gamma_i$.*

Assumption 3 (Super additivity)**.** *The individual valuation function $v(\boldsymbol{x}, \theta_j)$ is super additive with respect to $x_i, i = 0, \ldots, n-1$, i.e.,*

$$v(\boldsymbol{x}, \theta_j) \geq \sum_{i=0}^{n-1} v(x_i, \theta_j), \forall \boldsymbol{x}, \theta_j. \tag{1}$$

Assumption 4 (Decreasing cross marginal returns)**.** *The marginal return of one producer's input resources decreases when other producers contribute more input resources, i.e., for all $i \in N, j \in M, \theta_j$ and $x_i \geq x_i'$, $\boldsymbol{x}_{-i} \geq \boldsymbol{x}'_{-i}$, the following inequality holds:*

$$\begin{aligned} &v((x_i, \boldsymbol{x}_{-i}), \theta_j) - v((x_i', \boldsymbol{x}_{-i}), \theta_j) \\ \leq &v((x_i, \boldsymbol{x}'_{-i}), \theta_j) - v((x_i', \boldsymbol{x}'_{-i}), \theta_j). \end{aligned} \tag{2}$$

Assumption 5 (Correlated and identifiable valuation types)**.** *The prior belief on valuation types, Prior($\boldsymbol{\theta}$), is identifiable and correlated, i.e., the identifiability condition (Kosenok and Severinov 2008) and Crémer-McLean condition (Crémer and McLean 1985) hold for all consumers.*

The economic meaning of super additivity is that cooperative production brings synergies, i.e., the value created by the production coalition is higher than the total value created by independent producers. The rationality of Assumption 4 comes from the law of diminishing marginal returns: when many resources have already been involved in a production process, the marginal return brought by an additional unit of input resources decreases. As an example, the following individual valuation function satisfies all these four assumptions:

$$v(\boldsymbol{x}, \theta_j) = \theta_j \sqrt{n \sum_{k=0}^{n-1} z(x_k)^2}, \tag{3}$$

where $z(x_i)$ is an arbitrary smooth and increasing function (e.g., the Cobb-Douglas function). Assumption 5 is introduced to guarantee full consumer surplus, so that we can derive the income of the coalition $-\sum_{j=n}^{n+m-1} p_j$ from individual valuation functions, i.e., $-\sum_{j=n}^{n+m-1} p_j = \sum_{j=n}^{n+m-1} v(\boldsymbol{x}, \theta_j)$, because we have the following theorem:

Theorem 1 (Crémer-McLean Theorem)**.** *There exists an interim individually rational and ex-post budget balanced Bayesian mechanism that extracts full consumer surplus if Prior($\boldsymbol{\theta}$) is identifiable and Crémer-McLean condition holds for all consumers.*

Proofs of Theorem 1 can be found in (Crémer and McLean 1985) and (Kosenok and Severinov 2008). We can construct such a demand-side *Crémer-McLean mechanism* by following the constructive proof of Lemma A3 in (Kosenok and Severinov 2008) or by automated mechanism design techniques (Albert, Conitzer, and Lopomo 2015). We will not go into further detail here because we focus on the supply-side game. For theoretical analyses in this paper, we take as a given that the production coalition uses Crémer-McLean mechanism to extract full consumer surplus. This also guarantees the reported valuation type $\hat{\theta}_j$ equals the true valuation type θ_j for all consumer $j \in M$. Theorem 1 transfers the ex-post weak budget balance constraint $\sum_{l=0}^{n+m-1} p_l \leq 0$ to the following inequality:

$$\sum_{i=0}^{n-1} p_i \leq \sum_{j=n}^{n+m-1} v(\boldsymbol{x}, \theta_j). \tag{4}$$

This decouples the supply-side mechanism design problem from the demand-side problem.

The Procurement Auction

As a counterpart of Crémer-McLean mechanism which is optimal on the demand side, we introduce an optimal supply-side procurement auction in this section. This proposed procurement auction, accompnied by the demand-side Crémer-McLean mechanism, maximizes social surplus by incentivizing producers to truthfully report their capacity limits and type parameters. This procurement auction consists of four steps, of which the soundness will be proved in the next section.

Step 1. Producers bid on capacity limits and cost types

As the first step, every producer submits a sealed bid for their respective capacity limits and cost types. The reported capacity limit $\hat{x}_i$ is the maximum resources that producer i is willing to offer to the coalition. It may differ from the true capacity limit $\bar{x}_i$. Similarly, the reported cost type $\hat{\gamma}_i$ may differ from the true cost type γ_i.

Step 2. The coordinator chooses the optimal acceptance ratios

Then, the coalition coordinator decides how many input resources to accept from each producer. It chooses $x_i \leq \hat{x}_i, i = 0, \ldots, n-1$ that maximize the social surplus, constrained by reported capacity limits and based on reported type parameters. Equivalently, the coordinator calculates the optimal *acceptance ratio* $\eta_i \in [0,1]^{\dim(x_i)} = x_i \oslash \hat{x}_i$ such that $x_i = \hat{x}_i \odot \eta_i$, where $\odot$ and $\oslash$ denote the element-wise multiplication and division respectively, and $[0,1]$ denotes the interval between 0 and 1. The economic meaning of η_i is the ratio of input resources accepted by the coalition to those offered by the producer.

The optimal acceptance ratios $(\eta_0^*, \ldots, \eta_{n-1}^*) = \boldsymbol{\eta}^*$ are calculated according to the following formula:

$$\boldsymbol{\eta}^* = \operatorname{argmax}_{\boldsymbol{\eta} \in [0,1]^{\dim(x_i) \times n}} \{S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})\} \tag{5}$$
$$= \operatorname*{argmax}_{\boldsymbol{\eta} \in [0,1]^{\dim(x_i) \times n}} \sum_{j=n}^{n+m-1} v(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}, \theta_j) - \sum_{i=0}^{n-1} c_i(\hat{x}_i \odot \eta_i, \hat{\gamma}_i).$$

Because different $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ results in different $\boldsymbol{\eta}^*$, $\boldsymbol{\eta}^*$ is written as $\boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$. Correspondingly, the maximum social surplus is denoted by $S^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) = \sum_{j=n}^{n+m-1} v(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\theta}_j) - \sum_{i=0}^{n-1} c_i(\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_i)$. It is worth noting that although $S^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ and $S(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ both represent social surplus, they are different functions. The first parameter $\boldsymbol{x}$ in $S(\cdot)$ is the accepted input resources, whereas the first parameter $\hat{\boldsymbol{x}}$ in $S^*(\cdot)$ is the reported capacity limits. $\boldsymbol{x}$ and $\hat{\boldsymbol{x}}$ are related by $\boldsymbol{x} = \hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*$.

Step 3. Producers contribute accepted input resources to the production coalition

In this step, producers are required to contribute $\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*$ units of input resources to the production coalition. Since in the first step, producer i has promised to offer at most $\hat{x}_i$ units of input resources, if it cannot contribute $\hat{x}_i \odot \eta_i^* \leq \hat{x}_i$, we impose a high punishment on it. With the contributed input resources, producers collaboratively produce the output virtual product, bringing value $v(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j)$ to consumer $j \in M$.

Step 4. The coordinator makes transfer payments to participants according to the PVCG sharing rule

In this final step, the coordinator pays producers according to the PVCG sharing rule. The PVCG payment

$$p_i(\cdot) = \tau_i(\cdot) + h_i^*(\cdot) \tag{6}$$

is composed of two parts, the *VCG payment* τ_i and the *optimal adjustment payment* h_i^*. The VCG payment is designed to induce truthfulness, i.e., the reported capacity limits $\hat{\boldsymbol{x}}$ and reported cost type $\hat{\boldsymbol{\gamma}}$ are equal to the true capacity limits $\bar{\boldsymbol{x}}$ and true cost type $\boldsymbol{\gamma}$. The adjustment payment is optimized so that ex-post individual rationality and ex-post weak budget balancedness can also be attained.

With $\boldsymbol{\eta}^*$ calculated in Step 2, the VCG payment τ_i to producer i is:

$$\begin{aligned}
\tau_i &= S^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}_{-i}) - S_{-i}^*(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}}) \\
&\quad + c(\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_i) \\
&= \sum_{j=n}^{n+m-1} [v(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\theta}_j) - v(\hat{\boldsymbol{x}}_{-i} \odot \boldsymbol{\eta}^{-i*}(\\
&\quad \hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}}), \hat{\theta}_j)] - \sum_{k=0, \neq i}^{n-1} [c(\hat{x}_k \odot \eta_k^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_k) \\
&\quad - c(\hat{x}_k \odot \eta_k^{-i*}(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_k)],
\end{aligned} \tag{7}$$

where $(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i})$ denotes the reported capacity limits and the reported cost types excluding producer i. $\boldsymbol{\eta}^{-i*}$ and $S_{-i}^*(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$ are the corresponding optimal acceptance ratios and maximum social surplus. Note that $\boldsymbol{\eta}^{-i*}$ is different from $\boldsymbol{\eta}_{-i}^*$: the former maximizes $S(\hat{\boldsymbol{x}}_{-i} \odot \boldsymbol{\eta}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$, whereas the latter is the component of $\boldsymbol{\eta}^*$ that maximizes $S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$. $\boldsymbol{\tau} = (\tau_0, \ldots, \tau_{n-1})$ is a function of $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$, written as $\boldsymbol{\tau}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$.

The *adjustment payment* $h_i(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$ is a function of $(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$. The optimal adjustment payments $(h_0^*(\cdot), \ldots, h_{n-1}^*(\cdot)) = \boldsymbol{h}^*(\cdot)$ are determined by solving the following *functional equation* (a type of equation in which the unknowns are functions instead of variables; refer to (Rassias 2012) for more details):

$$\begin{aligned}
&\sum_{i=0}^{n-1} \mathrm{ReLu}[-(S^*(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S_{-i}^*(\boldsymbol{x}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})) \\
&- h_i(\boldsymbol{x}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})] + \mathrm{ReLu}\{\sum_{i=0}^{n-1} [(S^*(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \\
&- S_{-i}^*(\boldsymbol{x}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})) + h_i(\boldsymbol{x}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})] \\
&- S^*(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})\} \equiv 0, \; \forall (\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \mathrm{supp}(\mathrm{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})),
\end{aligned} \tag{8}$$

where $\mathrm{supp}(\mathrm{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}))$ is the *support* of the *prior distribution* $\mathrm{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ of the true parameters $(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ estimated by the coalition coordinator. Support is a terminology from *measure theory*, defined by $\mathrm{supp}(\mathrm{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})) = \{(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}) | \mathrm{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}) > 0\}$. In general, there is no closed-form solution to Eq. 8, so we employ neural network techniques to learn the solution. We will go into more details later.

Theoretical Analyses

Theoretical analyses in this section are organized through the following strand:

1. First (in Proposition 1 and Proposition 2), we prove that for arbitrary $h_i(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$, $i = 0, \ldots, n-1$, the PVCG payments guarantee supply-side truthfulness (dominant strategy incentive compatibility) and maximize social surplus (allocative efficiency).
2. Second (in Proposition 3 and Proposition 4), given that Crémer-McLean Theorem (Theorem 1) holds on the demand side, we derive two ineqaulity cosntraints on the adjustment payment $h_i(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$, which are sufficient and necessary conditions for ex-post individual rationality and ex-post weak budget balancedness. We show that these constraints can be transformed into Eq. 8, which is equivalent to a minimization problem that can be solved by neural network methods.
3. Lastly (in Theorem 2 and Corollary 4), we prove the existence of at least one solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8. The PVCG payment corresponding to this solution attains truthfulness, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balancedness simultaneously on the supply side.

First, we prove that for arbitrary $h_i(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}})$, the payment $p_i(\cdot) = \tau_i(\cdot) + h_i(\cdot)$ encourages all producers to report their capacity limits and cost types truthfully.

Proposition 1 (Dominant strategy incentive compatibility)**.** *For every producer i, truthfully reporting its capacity limit $\bar{x}_i$ and cost type γ_i is its dominant strategy, i.e.,*

$$\begin{aligned}&p_i((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}), (\gamma_i, \hat{\boldsymbol{\gamma}}_{-i}), \hat{\boldsymbol{\theta}})\\ &- c(\bar{x}_i \odot \eta_i^*((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}), (\gamma_i, \hat{\boldsymbol{\gamma}}_{-i}), (\theta_i, \hat{\boldsymbol{\theta}}_{-i})), \gamma_i)\\ &\geq p_i(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) - c_i(\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \gamma_i)\\ &\quad, \forall i \in N, x_i, \hat{\boldsymbol{x}}, \gamma_i, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}. \end{aligned} \tag{9}$$

Proof. We differentiate the cases where $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) > \bar{x}_i$ with those where $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \leq \bar{x}_i$.

(1) When $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) > \bar{x}_i$,
it is impossible for producer i to contribute the accepted amount of input resources $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ because this exceeds its capacity limit. In this case, producer i suffers a high punishment, i.e., $p_i(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ is a large negative number. Hence, the right side of Eq. 9 becomes extremely negative and Eq. 9 holds. This shows a rational producer will not choose $\hat{x}_i$ such that $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) > \bar{x}_i$.

(2) When $\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \leq \bar{x}_i$,
we aim to prove that by truthfully reporting $\bar{x}_i$, γ_i, and θ_i, the utility of producer i is at least the same as before.

By definition,
$S^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) = \max_{\boldsymbol{\eta} \in [0,1]^{\dim(x_i)\times n}} \{\sum_{j=n}^{n+m-1} v(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}, \hat{\theta}_j) - \sum_{i=0}^{n-1} c(\hat{x}_i \odot \eta_i, \hat{\gamma}_i)\}, \forall \hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}$. We substitute $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ with $((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}), (\gamma_i, \hat{\boldsymbol{\gamma}}_{-i}), \hat{\boldsymbol{\theta}})$ and get

$$\begin{aligned}&S^*((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}), (\gamma_i, \hat{\boldsymbol{\gamma}}_{-i}), \hat{\boldsymbol{\theta}})\\ \geq &\sum_{j=n}^{n+m-1} v((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}, \theta_j) - c(\bar{x}_i \odot \eta_i, \gamma_i)\\ &- \sum_{k=0,\neq i}^{n} c(\hat{x}_k \odot \eta_k, \hat{\gamma}_k), \forall \boldsymbol{\eta} \in [0,1]^{\dim(x_i)\times n}. \end{aligned} \tag{10}$$

In particular, for $\boldsymbol{\eta} = (\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \oslash \bar{x}_i, \boldsymbol{\eta}_{-i}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \boldsymbol{\theta})) \in [0,1]^{\dim(x_i)\times n}$, Eq. 10 holds. Therefore,

$$\begin{aligned}&S^*((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}), (\gamma_i, \hat{\boldsymbol{\gamma}}_{-i}), \hat{\boldsymbol{\theta}})\\ \geq &\sum_{j=n}^{n+m-1} [v((\bar{x}_i, \hat{\boldsymbol{x}}_{-i}) \odot (\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \oslash \bar{x}_i\\ &, \boldsymbol{\eta}_{-i}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})), \hat{\theta}_j) - \sum_{k=0,\neq i}^{n-1} c(\hat{x}_k \odot \eta_k^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_k)\\ &- c(\bar{x}_i \odot \hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \oslash \bar{x}_i, \gamma_i)\\ =&S^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})\\ &+ c(\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\gamma}_i) - c(\hat{x}_i \odot \eta_i^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \gamma_i). \end{aligned} \tag{11}$$

Adding $h_i(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}}_{-i}) - S_{-i}^*(\hat{\boldsymbol{x}}_{-i}, \hat{\boldsymbol{\gamma}}_{-i}, \hat{\boldsymbol{\theta}}_{-i})$ to both sides of Eq. 11 and substituting $p_i(\cdot) + v(\cdot) - c(\cdot) = S^*(\cdot) - S_{-i}^*(\cdot) + h(\cdot)$, we get Eq. 9. ☐

From Proposition 1, we know reported parameters equal true parameters, i.e., $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) = (\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$. Therefore, we can use these two sets of parameters interchangeably in the remaining parts of this paper.

Proposition 2 (Ex-post social surplus maximization / allocative efficiency)**.** *PVCG maximizes social surplus ex post.*

Proof. Suppose $\boldsymbol{x}^{**} = \text{argmax}_{\boldsymbol{x} \leq \bar{\boldsymbol{x}}}\{S(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta})\}$ and $S^{**} = S(\boldsymbol{x}^{**}, \boldsymbol{\gamma}, \boldsymbol{\theta})$. We aim to prove that PVCG results in ex-post social surplus no less than S^{**}.

By definition of $\boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$,

$$S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}) \geq S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \forall \boldsymbol{\eta}. \tag{12}$$

Incentive compatibility guarantees $\hat{\boldsymbol{x}} = \bar{\boldsymbol{x}}$, $\hat{\boldsymbol{\gamma}} = \boldsymbol{\gamma}$, and $\hat{\boldsymbol{\theta}} = \boldsymbol{\theta}$. Therefore,

$$S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \boldsymbol{\gamma}, \boldsymbol{\theta}) \geq S(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \forall \boldsymbol{\eta}. \tag{13}$$

Particularly, Eq. 13 holds for $\boldsymbol{\eta} = \boldsymbol{x}^{**} \oslash \bar{\boldsymbol{x}} \in [0,1]^{\dim(x_i)\times n}$, i.e.,

$$\begin{aligned}S(\hat{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}), \boldsymbol{\gamma}, \boldsymbol{\theta}) &\geq S(\bar{\boldsymbol{x}} \odot \boldsymbol{x}^{**} \oslash \bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})\\ &= S(\boldsymbol{x}^{**}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = S^{**}. \end{aligned} \tag{14}$$

The left side of Eq. 14 is the ex-post social surplus achieved by PVCG, while the right side is the maximum social surplus across all possible $\boldsymbol{x} \leq \bar{\boldsymbol{x}}$, given $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$. Hence, the right side is also no less than the left side. Therefore, the left side equals the right side. ☐

Furthermore, given that incentive compatibility has been proved in Proposition 1 and Theorem 1, the following two propositions provide sufficient and necessary conditions for ex-post individual rationality and ex-post weak budget balancedness.

Proposition 3 (Condition for ex-post individual rationality)**.** *PVCG is ex-post individual rational (IR) for all producers i.f.f. the true capacity limits $\bar{\boldsymbol{x}}$, the true type parameters $\boldsymbol{\gamma}, \boldsymbol{\theta}$, and the adjustment payments $\boldsymbol{h}(\cdot)$ satisfy*

$$\begin{aligned}&h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})\\ &\quad \geq -[S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S_{-i}^*(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})], \forall i \in N. \end{aligned} \tag{15}$$

Proof. According to truthfulness proved in Proposition 1 and Theorem 1, we use $\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}$ to substitute $\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}$ in Eq. 6 and Eq. 7. Then, the ex-post utility of producer i becomes

$$\begin{aligned}u_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) &= p_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - c(\bar{x}_i \odot \eta_i^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \gamma_i)\\ &= S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S_{-i}^*(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}) + h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}). \end{aligned} \tag{16}$$

Ex-post IR requires $u_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \geq 0, \forall i$, which is equivalent to the inequality in Eq. 15. ☐

Proposition 4 (Condition for ex-post weak budget balancedness)**.** *PVCG is ex-post weakly budget balanced (WBB) on the supply side i.f.f. the true capacity limits $\bar{\boldsymbol{x}}$, the true type parameters $\boldsymbol{\gamma}, \boldsymbol{\theta}$, and the adjustment payments $\boldsymbol{h}(\cdot)$ satisfy*

$$\begin{aligned}&\sum_{i=0}^{n-1} h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}) \leq S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})\\ &\quad - \sum_{i=0}^{n-1} [S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S_{-i}^*(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})]. \end{aligned} \tag{17}$$

Proof. According to truthfulness proved in Proposition 1 and Theorem 1, we use $\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}$ to substitute $\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}}$ in Eq. 6 and Eq. 7. Then, the ex-post total payment to all producers is

$$\begin{aligned}
&\sum_{i=0}^{n-1} p_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = \sum_{i=0}^{n-1} [\tau_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) + h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})] \\
&= \sum_{i=0}^{n-1} [S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}) \\
&+ c(\bar{x}_i \odot \eta_i^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \gamma_i) + h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})]. \qquad (18)
\end{aligned}$$

As explained before, Theorem 1 transforms the WBB condition to $\sum_{i=0}^{n-1} p_i(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \leq \sum_{j=n}^{n+m-1} v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \theta_j)$. This inequality, together with the definition $S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = \sum_{j=n}^{n+m-1} v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta_i^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \gamma_i)$, transforms Eq. 18 to Eq. 17. □

Then, we prove Eq. 8 is a sufficient and necessary confition for Proposition 3 and Proposition 4.

Corollary 1. *Under PVCG, a sufficient and necessary condition for ex-post IR and ex-post WBB to coexist on the supply side is*

$$LOSS = Loss1 + Loss2 = 0, \qquad (19)$$

where

$$\begin{aligned}
Loss1 = \sum_{i=0}^{n-1} ReLu[&-(S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})) \\
&- h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})] \quad \textit{and} \qquad (20) \\
Loss2 = ReLu[\sum_{i=0}^{n-1} &[(S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})) \\
&+ h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})] - S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})]. \qquad (21)
\end{aligned}$$

Proof. $Loss1 \geq 0, Loss2 \geq 0, \forall \bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}$. $Loss1 = 0$ i.f.f. Eq. 15 holds; $Loss2 = 0$ i.f.f. Eq. 17 holds. □

From Corollary 1, we know that if the adjustment payments $\boldsymbol{h}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ are such chosen that Eq. 19 holds for true capacity limits $\bar{\boldsymbol{x}}$ and true type parameters $\boldsymbol{\gamma}, \boldsymbol{\theta}$, then PVCG attains IR and WBB simultaneously. However, these true parameters are unknown to the coalition coordinator a priori, we need to find the *optimal adjustment payment function* $\boldsymbol{h}^*(\cdot)$ such that for all possible $\bar{\boldsymbol{x}}$ and $\boldsymbol{\gamma}$ drawn from their respective prior distributions, Eq. 19 holds. This is equivalent to solve the functional equation in Eq. 8. When such a functional solution exists, it minimizes the expected value of $LOSS = Loss1 + Loss2$.

Corollary 2. *The solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8, if existing, is also a solution to the following minimization problem:*

$$\boldsymbol{h}^*(\cdot) = argmin_{\boldsymbol{h}(\cdot)} \mathbb{E}_{(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})}\{LOSS\}, \qquad (22)$$

where the expectation is over the prior distribution of $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$.

Proof. For the solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8, $LOSS \equiv 0, \forall (\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \text{supp}(\text{Prior}(\boldsymbol{x}, \boldsymbol{\gamma}, \boldsymbol{\theta}))$. Therefore, $\mathbb{E}_{(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})}\{LOSS\} = 0$.

For all other function $\boldsymbol{h}(\cdot)$, since $LOSS \geq 0$, $\forall (\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$, we have $\mathbb{E}_{(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})}\{LOSS\} \geq 0$. □

From Corollary 2, we can find the functional solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8 by minimizing the expected $LOSS$. The remaining problem is to prove the existence of such a solution. The following theorem holds.

Theorem 2. *The following inequality is a sufficient and necessary condition for the existence of $\boldsymbol{h}^*(\cdot)$ such that the PVCG payments $\boldsymbol{p}(\cdot) = \boldsymbol{\tau}(\cdot) + \boldsymbol{h}^*(\cdot)$ satisfy WBB and IR for all $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in$ supp(Prior$(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$).*

$$\begin{aligned}
&\sum_{i=0}^{n-1} [S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*((\min \bar{x}_i, \boldsymbol{x}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta})] \\
&\leq S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \forall (\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \textit{supp}(\textit{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})), \quad (23)
\end{aligned}$$

where $\min \bar{x}_i$ and $\max \gamma_i$ are the extreme values of $\bar{\boldsymbol{x}}$ and $\boldsymbol{\gamma}$ on their support set supp(Prior$(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$).

In order to prove Theorem 2, we introduce the following lemma first.

Lemma 1. *The maximum social surplus monotonically increases with $\bar{x}_i$ and monotonically decreases with γ_i for every producer i.*

Proof. Suppose $\bar{\boldsymbol{x}}' \geq \bar{\boldsymbol{x}}$; then, by definition,

$$S^*(\bar{\boldsymbol{x}}', \boldsymbol{\gamma}, \boldsymbol{\theta}) \geq S(\bar{\boldsymbol{x}}' \odot \boldsymbol{\eta}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \forall \boldsymbol{\eta} \in [0, 1]^{\dim(x_i) \times n}. \quad (24)$$

Particularly, Eq. 13 holds for $\boldsymbol{\eta} = \boldsymbol{\eta}^*(\bar{\boldsymbol{x}}, \boldsymbol{\eta}, \boldsymbol{\theta}) \odot \bar{\boldsymbol{x}} \oslash \bar{\boldsymbol{x}}' \in [0, 1]^{\dim(x_i) \times n}$, i.e.,

$$\begin{aligned}
S^*(\bar{\boldsymbol{x}}', \boldsymbol{\gamma}, \boldsymbol{\theta}) &\geq S(\bar{\boldsymbol{x}}' \odot \boldsymbol{\eta}^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \odot \bar{\boldsymbol{x}} \oslash \bar{\boldsymbol{x}}', \boldsymbol{\gamma}, \boldsymbol{\theta}) \\
&= S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}). \qquad (25)
\end{aligned}$$

Therefore, $S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ increases with $\bar{x}_i$.

That $S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ decreases with γ_i is due to the *envelope theorem*, which results in

$$\frac{\partial S^*}{\partial \gamma_i}|_{\boldsymbol{\eta}=\eta^*} = \frac{\partial S}{\partial \gamma_i}|_{\boldsymbol{\eta}=\eta^*} = -\frac{\partial c_i}{\partial \gamma_i}|_{\boldsymbol{\eta}=\eta^*} \leq 0. \qquad (26)$$

□

Now we can prove Theorem 2 as follows.

Proof of Theorem 2. We prove sufficiency first.

If Eq. 23 holds, we aim to prove $h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}) \equiv -[S^*((\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})]$, $i = 0, \ldots, n-1$ satisfy Eq. 15 and 17 for all $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}))$.

In this case, all $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ satisfy Eq. 15 because of the monotonic property proved in Lemma 1, i.e.,

$$\begin{aligned}
&S^*((\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) \\
&\leq S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \forall \bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta} \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})). \quad (27)
\end{aligned}$$

Meanwhile, Eq. 17 becomes

$$\sum_{i=0}^{n-1}\{-[S^*((\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})]\} \le S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - \sum_{i=0}^{n-1}[S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})], \tag{28}$$

which can be deduced from Eq. 23.

To prove necessity, we set $\bar{\boldsymbol{x}} = (\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i})$ and $\boldsymbol{\gamma} = (\max \gamma_i, \boldsymbol{\gamma}_{-i})$ in Eq. 15 and get a necessary condition:

$$h_i(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta}) \ge -[S^*((\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) - S^*_{-i}(\bar{\boldsymbol{x}}_{-i}, \boldsymbol{\gamma}_{-i}, \boldsymbol{\theta})], \forall \bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta} \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})). \tag{29}$$

Eq. 29, together with Eq. 17, results in Eq. 23. □

There are some useful corollaries of Theorem 2.

Corollary 3. *The functional solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8 exists if the prior estimation on $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}$ is accurate enough.*

Proof. When the the prior estimation on $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma})$ is accurate enough, $\bar{x}_i$ approaches to $\min \bar{x}_i$ and γ_i approaches to $\max \gamma_i$); hence, the left side of Eq. 23 approaches 0, while the right side approaches a positive number. □

Corollary 4. *If the individual valuation function is super additive and with decreasing cross marginal returns, there exists at least one solution $\boldsymbol{h}^*(\cdot)$ to Eq. 8 such that dominant incentive compatibility, ex-post allocative efficiency, ex-post individual rationality, and ex-post weak budget balancedness are simultaneously attained on the supply side.*

Proof. According to Lemma 1,

$$S^*((\min \bar{x}_i, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) \tag{30}$$
$$\ge S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), (\max \gamma_i, \boldsymbol{\gamma}_{-i}), \boldsymbol{\theta}) = S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), \boldsymbol{\gamma}, \boldsymbol{\theta}).$$

Hence, a sufficient condition for Eq. 23 to hold in this case is

$$\sum_{i=0}^{n-1}[S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), \boldsymbol{\gamma}, \boldsymbol{\theta})] \le S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \quad \forall \bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta} \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})). \tag{31}$$

For arbitrary $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}))$, suppose $\boldsymbol{\eta}^*$ maximizes $S(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}, \boldsymbol{\gamma}, \boldsymbol{\theta})$. The corresponding optimal social surplus are denoted by $S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$, i.e.,

$$S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = S(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \boldsymbol{\gamma}, \boldsymbol{\theta}) = \sum_{j=n}^{n+m-1} v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta_i^*, \gamma_i). \tag{32}$$

We use η_i^* and $\boldsymbol{\eta}^*_{-i}$ to denote the components of $\boldsymbol{\eta}^*$.

According to the definition of $S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), \boldsymbol{\gamma}, \boldsymbol{\theta})$, we know

$$S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), \boldsymbol{\gamma}, \boldsymbol{\theta}) \ge S((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}, \boldsymbol{\gamma}, \boldsymbol{\theta}), \forall \boldsymbol{\eta}, \tag{33}$$

which is particularly true for $\boldsymbol{\eta} = \boldsymbol{\eta}^*$

According to Assumption 4 and 2,

$$\begin{aligned} & v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j) - v((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}^*, \theta_j) \\ =& v((\bar{x}_i \odot \eta_i^*, \bar{\boldsymbol{x}}_{-i} \odot \boldsymbol{\eta}^*_{-i}), \theta_j) - v((\mathbf{0}, \bar{\boldsymbol{x}}_{-i} \odot \boldsymbol{\eta}^*_{-i}), \theta_j) \\ \le& v((\bar{x}_i \odot \eta_i^*, \mathbf{0}, \dots, \mathbf{0}), \theta_j) - v((\mathbf{0}, \mathbf{0}, \dots, \mathbf{0}), \theta_j) \\ =& v(\bar{x}_i \odot \eta_i^*, \theta_j), \forall i \in N, j \in M. \end{aligned} \tag{34}$$

Then, we can derive

$$\begin{aligned} & S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) = S(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \boldsymbol{\gamma}, \boldsymbol{\theta}) \\ =& \sum_{j=n}^{n+m-1} v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta_i^*, \gamma_i) \\ \ge& \sum_{j=n}^{n+m-1}\sum_{i=0}^{n-1} v(\bar{x}_i \odot \eta_i^*, \theta_j) - \sum_{i=0}^{n-1} c(\bar{x}_i \odot \eta_i^*, \gamma_i) \\ \ge& \sum_{i=0}^{n-1}\{ \sum_{j=n}^{n+m-1} [v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j) - v((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}^*, \theta_j)] \\ & - c(\bar{x}_i \odot \eta_i^*, \gamma_i)\} \\ =& \sum_{i=0}^{n-1}[\sum_{j=n}^{n+m-1} v(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \theta_j) - \sum_{k=0}^{n-1} c(\bar{x}_k \odot \eta_k^*, \gamma_k)] \\ & - \sum_{i=0}^{n-1}[\sum_{j=n}^{n+m-1} v((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}^*, \theta_j) \\ & \qquad - c(\mathbf{0} \odot \eta_i^*, \gamma_i) - \sum_{k=0,\neq i}^{n-1} c(\bar{x}_k \odot \eta_k^*, \gamma_k)] \\ =& \sum_{i=0}^{n-1}[S(\bar{\boldsymbol{x}} \odot \boldsymbol{\eta}^*, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}) \odot \boldsymbol{\eta}^*, \boldsymbol{\gamma}, \boldsymbol{\theta})] \\ \ge& \sum_{i=0}^{n-1}[S^*(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) - S^*((\mathbf{0}, \bar{\boldsymbol{x}}_{-i}), \boldsymbol{\gamma}, \boldsymbol{\theta})], \end{aligned} \tag{35}$$

where the first ineqaulity is due to super additivity, the second inequality is a result from Eq. 34, and the third inequality is from Eq. 32 and 33. Eq. 35 is true for arbitrary $(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}) \in \text{supp}(\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta}))$, so we get Eq. 31, which is a sufficient condition for the existence of $\boldsymbol{h}^*(\cdot)$ as a solution to 8. □

Lastly, it is worth pointing out even for those cases where there is no solution to Eq. 8 (i.e., Assuption 3-5 do not hold), an optimal $\boldsymbol{h}^*$ can still be found by minimizing the expected $LOSS$ in Eq. 22. Such an optimal $\boldsymbol{h}^*$ guarantees the expected deviation from the IR and WBB constraints are minimized.

Learning the Optimal Adjustment Payments

Corollary 2 informs us that we can learn the optimal adjustment payments $\boldsymbol{h}^*(\cdot)$ by minimizing the expected LOSS in Eq. 22. Also, we know neural networks can approximate arbitrary continuous functions to arbitrary precisions (Funahashi 1989). Therefore, we construct n neural networks

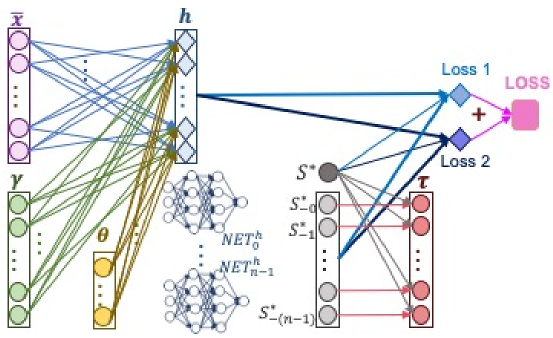

Figure 1: The structure of the composite neural network of PVCG

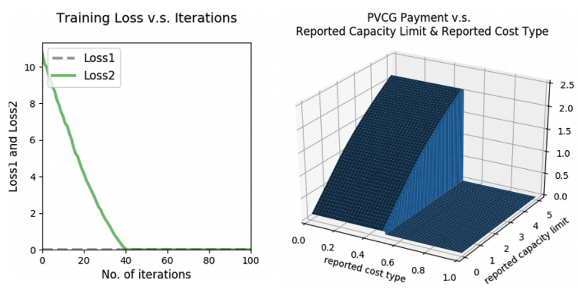

Figure 2: Training loss v.s. iterations (left) and PVCG payment v.s. reported capacity limit & reported cost type (right)

$\text{NET}_i^h, i \in N$ to approximate $h_i(\cdot), i \in N$. Output nodes of these n networks, denoted by $\text{NET}_i^h.o, i \in N$, are combined into a single *composite neural network* in Figure 1 with the following loss function:

$$\text{LOSS} = \frac{1}{T}\sum_{t=0}^{T-1}\{\sum_{i=0}^{n-1}\text{ReLu}[-(S^{*t} - S^{*t}_{-i}) - \text{NET}_i^h.o] + \text{ReLu}[\sum_{i=0}^{n-1}([(S^{*t} - S^{*t}_{-i}) + \text{NET}_i^h.o] - S^{*t})], \quad (36)$$

where each sample $(\bar{\boldsymbol{x}}^t, \boldsymbol{\gamma}^t, \boldsymbol{\theta}^t)$ is drawn from their prior distribution $\text{Prior}(\bar{\boldsymbol{x}}, \boldsymbol{\gamma}, \boldsymbol{\theta})$ and T is the sample size. For the tth sample, $\boldsymbol{\tau}^t = \boldsymbol{\tau}(\bar{\boldsymbol{x}}^t, \boldsymbol{\gamma}^t, \boldsymbol{\theta}^t)$, $S^{*t} = S^*(\bar{\boldsymbol{x}}^t, \boldsymbol{\gamma}^t, \boldsymbol{\theta}^t)$, and $S^{*t}_{-i} = S^*(\bar{\boldsymbol{x}}^t_{-i}, \boldsymbol{\gamma}^t_{-i}, \boldsymbol{\theta}^t_{-i})$. Since we only need synthetic data to train this network, we can generate as many data as needed. As a result, LOSS can be minimized to its theoretical minimum almost perfectly in experiments.

To illustrate the effectiveness of this neural network method and to show the correctness of Corollary 4, we learned the adjustment payments for a hypothetical scenario. We set the individual valuation functions and individual cost functions as follows:

$$v(\boldsymbol{x}) = \theta_i\sqrt{n(\sum_{k=0}^{n-1} x_k)} \text{ and } c_i(x_i, \gamma_i) = \gamma_i x_i, i \in N. \quad (37)$$

Note that the individual valuation function in Eq. 37 is a special case of that in Eq. 3, where $z(x_i)$ is set to be $z(x_i) = \sqrt{x_i}$. We report the experiment results for $n = 10, m = 2$, $\text{Prior}(\bar{x}_i) = \text{Uniform}[0, 5], i \in N$, $\text{Prior}(\gamma_i) = \text{Uniform}[0, 1], i \in N$, and $\text{Prior}(\theta_j) = [0, 1], j \in M$. We let $\text{NET}_i^h, i \in N$ each have three 10-dimensional hidden layers.

The loss curve is shown in the left figure of Fig. 2. The training loss fast converges to 0 after 40 iterations, as expected. After we obtain the trained networks $[\text{NET}_i^h], i \in N$, we can use trained networks to calculate PVCG payments $\boldsymbol{p}(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$ for any reported $(\hat{\boldsymbol{x}}, \hat{\boldsymbol{\gamma}}, \hat{\boldsymbol{\theta}})$. For illustration, we draw p_0, the payment to participant 0, with respect to $\hat{x}_0$ and $\hat{\gamma}_0$ in the right figure in Fig. 2, fixing parameters of other participants at $\hat{x}_i \equiv 2.5, \hat{\gamma}_i \equiv 0.5, i \in N, \neq 0, \hat{\theta}_j \equiv 0.5, j \in M$.

We can see that p_0 increases with $\bar{x}_0$. This aligns well with our expectation that the higher the capacity limit a producer reports, the more input resources are contributed by this producer; hence, it receives higher payments. Also, p_0 remains constant with γ_0 when γ_0 is below a threshold and sharply drops to around 0 when γ_0 passes the threshold. This implies that the payment to a producer should only be affected by its contribution to the cooperative production process rather than its cost, but if a producer's cost is too high, the optimal social choice is to exclude this producer from the production coalition and thus pay it nothing.

Conclusions and Future Work

In this paper, we presented the Procurement-VCG (PVCG) mechanism for incentivizing producers to collaboratively produce some valuable virtual products. PVCG incentivizes capacitated producers to truthfully report their capacity limits and their cost types, based on which we optimize for social surplus and guarantee ex-post allocative efficiency. With some reasonable assumptions, we prove that PVCG is ex-post individually rational and ex-post weakly budget balanced. To our knowledge, PVCG is among the very few mechanisms in the mechanism design literature that attain these four objectives simultaneously.

Since this paper focuses on theoretical analyses, we reported limited experimental results. When the number of producers increase, we require more advanced algorithms to learn the PVCG payments efficiently. Besides, for non-analytic individual valuation functions and individual cost functions, calculating the optimal social surplus is computationally expensive as well. These computational issues are beyond the scope of this paper and left for future research. Also, in order to apply PVCG to scenarios such as federated learning, some special-purpose engineering designs (e.g., a sandbox) are required to measure the required parameters in our model, e.g., the input datasets and the output model. These implementation details will be discussed in our subsequent research. Moreover, in our following work, we will report simulation results that compare PVCG with other sharing rules for cooperative games, e.g., Shapley (Jia et al. 2019), Labour Union (Gollapudi et al. 2017), etc.

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